

✓ **21** Derivujte podle pravidel pro derivaci součinu, podílu.

$$h_1: y = x \cdot \sin x$$

$$h_2: y = (x^2 - 1) \cdot \sin x$$

$$h_3: y = \sin x \cdot \operatorname{tg} x$$

$$h_4: y = \frac{2x - 1}{x + 3}$$

$$h_5: y = \frac{x^2 + 2x}{1 - x^2}$$

$$h_6: y = \frac{\sin x + \cos x}{\sin x - \cos x}$$

✓ **22** Vypočítejte derivace složených funkcí.

$$✓ f_1(x) = (x^2 + 1)^6$$

$$✓ f_2(x) = \sqrt{4x^3 - x}$$

$$✓ f_3(x) = (\sqrt{2x^3 - 1} + 2)^8$$

$$✓ f_4(x) = \frac{1}{(3x^4 + x^2)^{10}}$$

$$✓ f_5(x) = \sqrt{x + \sqrt{5x}}$$

$$✓ f_6(x) = \cos(2x + 4)$$

$$✓ f_7(x) = \sin^2 x$$

$$✓ f_8(x) = \sin x^2$$

$$✓ f_9(x) = \frac{1}{\sin^3 x}$$

$$✓ f_{10}(x) = \sqrt{\cos 2x}$$

$$✓ f_{11}(x) = \sqrt[3]{\cos 2x + 2x}$$

$$✓ f_{12}(x) = \operatorname{tg} \left(3x - \frac{\pi}{4}\right)$$

$$✓ f_{13}(x) = \sqrt{\sin 3x + 5}$$

$$✓ f_{14}(x) = \ln(2x + 4)$$

$$✓ f_{15}(x) = \ln(3 \sin x - 8)$$

$$✓ f_{16}(x) = e^{\sin x}$$

23 Je dána funkce $g: y = x^3 + 2x$. Vypočítejte $g'(0)$, $g'(1)$, $g'(-2)$.

24 Je dána funkce $f: y = 2x^3 - 2x + 1$. Určete, pro která $x \in \mathbb{R}$ platí:

a) $f'(x) = 0$

b) $f'(x) = 4$

c) $f'(x) = -5$

d) $f'(x) = f'(3)$

19.6 Tečna ke grafu funkce

25 Určete směrnici tečny ke grafu funkce $y = x^2$ v bodě $T[3; 9]$.

26 Napište rovnici tečny ke grafu funkce $y = f(x)$ v bodě T . Rovnici tečny uveďte v obecném tvaru.

a) $f_1(x) = x^2 - 2x$, $T[4; ?]$

b) $f_2(x) = 2x^4 + 8x$, $T[-1; ?]$

c) $f_3(x) = \frac{1}{x^2}$, $T\left[\frac{1}{2}; ?\right]$

d) $f_4(x) = \frac{2x - 1}{x + 1}$, $T[-2; ?]$

e) $f_5(x) = 2 \sin x$, $T[0; ?]$

f) $f_6(x) = x \operatorname{tg} x$, $T[0; ?]$

g) $f_7(x) = \frac{1 + x^3}{x - 1}$, $T[2; ?]$

h) $f_8(x) = \frac{\sin 2x + 1}{\cos x + \sin x}$, $T\left[\frac{\pi}{2}; ?\right]$

$$\mathbf{21} \quad h'_1(x) = \sin x + x \cos x, \quad h'_2(x) = 2x \sin x + x^2 \cos x - \cos x, \quad h'_3(x) = \sin x + \frac{\sin x}{\cos^2 x},$$

$$h'_4(x) = \frac{7}{(x+3)^2}, \quad h'_5(x) = \frac{2x^2+2x+2}{(1-x^2)^2}, \quad h'_6(x) = \frac{-2}{1-\sin 2x}.$$

$$\mathbf{22} \quad f'_1(x) = 12x(x^2 + 1)^5, \quad f'_2(x) = \frac{12x^2-1}{2\sqrt{4x^3-x}}, \quad f'_3(x) = \frac{24x^2(\sqrt{2x^3-1}+2)^7}{\sqrt{2x^3-1}},$$

$$f'_4(x) = \frac{-10(12x^3+2x)}{(3x^4+x^2)^{11}}, \quad f'_5(x) = \frac{2\sqrt{5x+5}}{4\sqrt{5x^2+5x}\sqrt{5x}}, \quad f'_6(x) = -2 \sin(2x + 4), \quad f'_7(x) = \sin 2x,$$

$$f'_8(x) = 2x \cos x^2, \quad f'_9(x) = \frac{-3 \cos x}{\sin^4 x}, \quad f'_{10}(x) = -\frac{\sin 2x}{\sqrt{\cos 2x}}, \quad f'_{11}(x) = \frac{2-2 \sin 2x}{3 \cdot \sqrt[3]{(\cos 2x+2x)^2}}, \quad f'_{12} =$$

$$= \frac{3}{\cos^2(3x-\frac{\pi}{4})}, \quad f'_{13}(x) = \frac{3 \cos 3x}{2\sqrt{\sin 3x+5}}, \quad f'_{14}(x) = \frac{1}{x+2}, \quad f'_{15}(x) = \frac{3 \cos x}{3 \sin x - 8},$$

$$f'_{16}(x) = e^{\sin x} \cdot \cos x.$$

$$\mathbf{23} \quad g'(0) = 2, \quad g'(1) = 5, \quad g'(-2) = 14.$$

$$\mathbf{24} \quad \text{a) } x \in \{\pm \frac{\sqrt{3}}{3}\}; \quad \text{b) } x \in \{\pm 1\}; \quad \text{c) } x \in \emptyset; \quad \text{d) } x \in \{-3\}.$$

19.6 Tečna ke grafu funkce

$$\mathbf{25} \quad k = 6. \quad \mathbf{26} \quad \text{a) } 6x - y - 16 = 0; \quad \text{b) } y + 6 = 0; \quad \text{c) } 16x + y - 12 = 0; \quad \text{d) } 3x - y + 11 = 0;$$

$$\text{e) } 2x - y = 0; \quad \text{f) } y = 0; \quad \text{g) } 3x - y + 3 = 0; \quad \text{h) } 2x + 2y - \pi - 2 = 0. \quad \mathbf{27} \quad \text{a) } T[1; 2];$$