① $z = 2(\cos 20° + i \sin 20°) \cdot 3(\cos 70° + i \sin 70°) =$

$= 6(\cos 90° + i \sin 90°) = 6(0 + i) = \underline{6i}$

② $z = \dfrac{6(\cos 80° + i \sin 80°)}{2(\cos 20° + i \sin 20°)} = 3(\cos 60° + i \sin 60°) =$

$= 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \underline{\frac{3}{2} + \frac{3\sqrt{3}}{2}i}$

③ $z = \dfrac{8\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)}{4\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)} = 2\left(\cos\left(\frac{7\pi}{6} - \frac{10\pi}{6}\right) + i \sin\left(-\frac{3\pi}{6}\right)\right)$

$= 2\left(\cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)\right) = 2(0 - i) = \underline{-2i}$

④ $z = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^5 = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right) =$

$2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \underline{1 - \sqrt{3}i}$

⑤ $z = \left(\sqrt{2}\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)\right)^7 = 2^{\frac{7}{2}}\left(\cos \frac{14\pi}{3} + i \sin 4\frac{2\pi}{3}\right) =$

$= 2^{\frac{7}{2}}\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) = 2^{\frac{6}{2}} \cdot 2^{\frac{1}{2}}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) =$

$= 2^3\sqrt{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -2^2\sqrt{2} + 2^2\sqrt{6}i = \underline{-4\sqrt{2} + 4\sqrt{6}i}$

Rozwiąż

① $2\bar{z} - 1 + i = z$

$2(a+bi) - 1 + i = a + bi$

$2a - 2bi - 1 + i = a + bi$

$a - 3bi = 1 - i$

$a = 1$ $\quad -3bi = -i$

$3b = 1$

$b = \frac{1}{3}$

$z = 1 + \frac{1}{3}i$

Zb: $L = 2(1 - \frac{1}{3}i) - 1 + i =$

$= 2 - \frac{2}{3}i - 1 + i = 1 + \frac{1}{3}i$

$P = 1 + \frac{1}{3}i$

$L = P$

② $z\bar{z} - z = \overline{6 - 2i}$

$(a+bi)(a-bi) - (a+bi) = 6 + 2i$

$a^2 - b^2 i^2 - a - bi = 6 + 2i$

$a^2 + b^2 - a - bi = 6 + 2i$

$a^2 + b^2 - a = 6$ $\quad -bi = 2i$

$a^2 + 4 - a = 6$ $\quad -b = 2$

$b = -2$

$a^2 - a - 2 = 0$

$(a-2)(a+1) = 0$

$a_1 = 2$

$a_2 = -1$

$z_1 = 2 - 2i$

$z_2 = -1 - 2i$

3) $|z| = 1 + 2i + z$

$\sqrt{a^2+b^2} = 1 + 2i + a + bi$

$\sqrt{a^2+b^2} = 1 + a$

$\sqrt{a^2+4} = 1 + a \quad /^2$

$a^2 + 4 = 1 + 2a + a^2$

$4 = 1 + 2a$

$3 = 2a$

$a = \frac{3}{2}$

$0 = 2i + bi$

$0 = 2 + b$

$b = -2$

$\boxed{z = \frac{3}{2} - 2i}$

zk: $L = \left|\frac{3}{2} - 2i\right| = \sqrt{\frac{9}{4} + 4} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

$P = 1 + 2i + \frac{3}{2} - 2i = \frac{5}{2}$

$L = P$

4) $z^2 + z + 1 = 0$

$z = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

$\boxed{\begin{array}{l} z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i \\ z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i \end{array}}$

5) $2z^2 - 4z + 5 = 0$

$z = \frac{4 \pm \sqrt{16 - 4 \cdot 5 \cdot 2}}{2 \cdot 2} = \frac{4 \pm \sqrt{4(4-10)}}{4} = \frac{4 \pm 2\sqrt{-6}}{4} = \frac{4 \pm 2i\sqrt{6}}{4}$

$z_1 = \frac{4}{4} - \frac{2}{4}\sqrt{6} i = 1 - \frac{\sqrt{6}}{2} i$

$z_2 = \frac{4}{4} + \frac{2}{4}\sqrt{6} i - 1 + \frac{\sqrt{6}}{2} i$

⑥ $z^2 - 2iz - 2 = 0$

$$z = \frac{2i \pm \sqrt{4i^2 - 4 \cdot 1 \cdot (-2)}}{2} = \frac{2i \pm \sqrt{-4+8}}{2} = \frac{2i \pm \sqrt{4}}{2}$$

$$= \frac{2i \pm 2}{2} = i \pm 1$$

$z_1 = i + 1$

$z_2 = i - 1$

⑦ $(3-4i)^2 - 2\bar{z} = z$

$9 - 24i + 16i^2 - 2(a-bi) = a+bi$

$9 - 24i - 16 - 2a + 2bi = a + bi$

$-7 - 24i = 3a - bi$

$-7 = 3a \qquad -24i = -bi$

$a = -\frac{7}{3} \qquad b = 24$

$z = -\frac{7}{3} + 24i$

⑧ $2z + 3\bar{z} = 5 + i$

$2(a+bi) + 3(a-bi) = 5 + i$

$2a + 2bi + 3a - 3bi = 5 + i$

$5a - bi = 5 + i$

$5a = 5 \qquad -bi = i$

$a = 1 \qquad b = -1$

$z = 1 - i$

9)

$z(\bar{z}-4)-1=8i$

$(a+bi)(a-bi-4)-1=8i$

$a^2-abi-4a+abi-b^2i^2-4bi-1=8i$

$a^2-4a+b^2-4bi-1=8i$

$-4bi=8i$

$b=-2$

$a^2-4a+b^2-1=0$

$a^2-4a+4-1=0$

$a^2-4a+3=0$

$(a-3)(a-1)=0$

$a_1=1$

$a_2=3$

$\boxed{z_1=1-2i \quad z_2=3-2i}$

10) $z^2-3z+10=0$

$z=\frac{3\pm\sqrt{9-4\cdot10\cdot1}}{2}=\frac{3\pm\sqrt{9-40}}{2}=\frac{3\pm\sqrt{-31}}{2}=\frac{3\pm i\sqrt{31}}{2}$

$z_1=\frac{3}{2}+\frac{\sqrt{31}}{2}i$

$z_2=\frac{3}{2}-\frac{\sqrt{31}}{2}i$

11) $z^2-6iz-8=0$

$z=\frac{6i\pm\sqrt{36i^2-4\cdot1\cdot(-8)}}{2\cdot1}=\frac{6i\pm\sqrt{-36+32}}{2}=$

$=\frac{6i\pm\sqrt{-4}}{2}=\frac{6i\pm2i}{2}$

$z_1=\frac{6i+2i}{2}=\frac{8i}{2}=4i$

$z_2=\frac{6i-2i}{2}=\frac{4i}{2}=2i$